A quadratic function, \( f(x) = ax^2 + bx \), is represented by the mapping diagram below.

1a. Use the mapping diagram to write down two equations in terms of \( a \) and \( b \). [2 marks]

**Markscheme**

\[
4a + 2b = 20 \\
a + b = 8 \quad (A1) \\
a - b = -4 \quad (A1) \quad (C2)
\]

**Note:** Award \((A1)(A1)\) for any two of the given or equivalent equations.

[2 marks]

1b. Find the value of \( a \). [1 mark]

**Markscheme**

\( a = 2 \) \( (A1)(\text{ft}) \)

[1 mark]

1c. Find the value of \( b \). [1 mark]

**Markscheme**

\( b = 6 \) \( (A1)(\text{ft}) \) \( (C2) \)

**Note:** Follow through from their (a).

[1 mark]

1d. Calculate the \( x \)-coordinate of the vertex of the graph of \( f(x) \). [2 marks]
Markscheme

\[ x = \frac{6}{2} \]  

(M1)

Note: Award (M1) for correct substitution in correct formula.

\[ = -1.5 \]  

(A1)(R)  

(C2)  

[2 marks]

The graph of \( y = 2x^2 - rx + q \) is shown for \(-5 \leq x \leq 7\).

2a. Write down the value of \( q \).  

[1 mark]

Markscheme

\( q = 4 \)  

(A1)  

(C1)  

[1 mark]

2b. The axis of symmetry is \( x = 2.5 \).

Find the value of \( r \).  

[2 marks]

Markscheme

\[ 2.5 = \frac{r}{4} \]  

(M1)  

\( r = 10 \)  

(A1)  

(C2)  

[2 marks]

2c. The axis of symmetry is \( x = 2.5 \).

Write down the minimum value of \( y \).  

[1 mark]
2d. The axis of symmetry is \( x = 2.5 \). Write down the range of \( y \).

**Markscheme**

\(-8.5 \leq y \leq 104 \)  

(A1)(ft) (A1)(ft) (C2)

Notes: Award (A1)(ft) for their answer to part (c) with correct inequality signs, (A1)(ft) for 104. Follow through from their values of \( q \) and \( r \).
Accept 104 ±2 if read from graph.

[2 marks]

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The following is the graph of the quadratic function \( y = f(x) \).

3a. Write down the solutions to the equation \( f(x) = 0 \).

**Markscheme**

\( x = 0, x = 4 \)  

(A1)(A1) (C2)

Notes: Accept 0 and 4.

[2 marks]

3b. Write down the equation of the axis of symmetry of the graph of \( f(x) \).
The equation \( f(x) = 12 \) has two solutions. One of these solutions is \( x = 6 \). Use the symmetry of the graph to find the other solution.

**Markscheme**

\[ x = 2 \quad (A1)(A1) \quad (C2) \]

**Note:** Award \((A1)\) for \( x = \) constant, \((A1)\) for 2.

\([2 \text{ marks}]\)

3c. The equation \( f(x) = 12 \) has two solutions. One of these solutions is \( x = 6 \). Use the symmetry of the graph to find the other solution.

**Markscheme**

\[ x = -2 \quad (A1) \quad (C1) \]

**Note:** Accept \(-2\).

\([1 \text{ mark}]\)

3d. The minimum value for \( y \) is \(-4\). Write down the range of \( f(x) \).

**Markscheme**

\[ y \geqslant -4 \quad (f(x) \geqslant -4) \quad (A1) \quad (C1) \]

**Notes:** Accept alternative notations.

Award \((A0)\) for use of strict inequality.

\([1 \text{ mark}]\)

The \( x \)-coordinate of the minimum point of the quadratic function \( f(x) = 2x^2 + kx + 4 \) is \( x = 1.25 \).

4a. (i) Find the value of \( k \).

\([4 \text{ marks}]\)

(ii) Calculate the \( y \)-coordinate of this minimum point.


**Markscheme**

(i) \( 1.25 = -\frac{k}{2(1.25)} \) \((M1)\)

OR

\( f'(x) = 4x + k = 0 \) \((M1)\)

**Note:** Award \((M1)\) for setting the gradient function to zero.

\( k = -5 \) \((A1)\) \((C2)\)

(ii) \( 2(1.25)^2 - 5(1.25) + 4 \) \((M1)\)

\( = 0.875 \) \((A1)(R)\) \((C2)\)

**Note:** Follow through from their \( k \).

**4 marks**

---

4b. Sketch the graph of \( y = f(x) \) for the domain \(-1 \leq x \leq 3\). \([2 \text{ marks}]\)
Consider the quadratic function \( y = f(x) \), where \( f(x) = 5 - x + ax^2 \).

5a. It is given that \( f(2) = -5 \). Find the value of \( a \).

\[ -5 = 5 - (2) + a(2)^2 \quad (MI) \]

Note: Award (MI) for correct substitution in equation.

\((a =) -2 \quad (A1) \quad (C2)\)

[2 marks]

5b. Find the equation of the axis of symmetry of the graph of \( y = f(x) \).

\[ x = \frac{-b}{2a} = -\frac{1}{4} \quad (A1)(A1)(ft) \quad (C2) \]

Notes: Follow through from their part (a). Award (A1)(A0)(ft) for “\( x = \) constant”. Award (A0)(A1)(ft) for \( y = -\frac{1}{4} \).

[2 marks]

5c. Write down the range of this quadratic function.

[2 marks]
6a. Write down the equation of the axis of symmetry.  

**Markscheme**

$x = 2$  


**Notes:** Award (AI) for “$x = \text{constant}$” (other than 2). Award (A0)(AI) for $y = 2$. Award (A0)(A1) for only seeing 2. Award (A0)(A0) for $2 = \frac{-b}{2a}$.

[2 marks]

6b. Sketch the graph of $y = f(x)$ on the axes below for $0 \leq x \leq 4$. Mark clearly on the sketch the points A, B, and C.  

[3 marks]
6c. The graph of \( y = f(x) \) intersects the \( x \)-axis for a second time at point D.

Write down the \( x \)-coordinate of point D.

**Markscheme**

3 (A1)(B) (C1)

Notes: (A0) for coordinates. Accept \( x = 3 \) or \( D = 3 \).

[1 mark]
Part of the graph of the quadratic function $f$ is given in the diagram below.

On this graph one of the $x$-intercepts is the point $(5, 0)$. The $x$-coordinate of the maximum point is 3. The function $f$ is given by $f(x) = -x^2 + bx + c$, where $b, c \in \mathbb{Z}$

7a. Find the value of $f(x)$

(i) $b$ ;

(ii) $c$ .

**Markscheme**

(i) $3 = \frac{-b}{2}$ (M1)

Note: Award (M1) for correct substitution in formula.

OR

$-1 + b + c = 0$

$-25 + 5b + c = 0$

$-24 + 4b = 0$ (M1)

Notes: Award (M1) for setting up 2 correct simultaneous equations.

OR

$-2x + b = 0$ (M1)

Notes: Award (M1) for correct derivative of $f(x)$ equated to zero.

$b = 6$ (A1) (C2)

(ii) $0 = -(5)^2 + 6 \times 5 + c$

$c = -5$ (A1) (B1) (C1)

Notes: Follow through from their value for $b$.

Notes: Alternatively candidates may answer part (a) using the method below, and not as two separate parts.

$(x - 5)(-x + 1)$ (M1)

$-x^2 + 6x - 5$ (A1)

$b = 6 \quad c = -5$ (A1) (C3)

[3 marks]
7b. The domain of \( f \) is \( 0 \leq x \leq 6 \).

Find the range of \( f \).

**Markscheme**

\(-5 \leq y \leq 4 \) \( (AI)(ft)(AI)(ft)(AI) \) \( (C3) \)

**Notes:** Accept \([-5, 4]\). Award \((AI)(ft)\) for \(-5\), \((AI)(ft)\) for 4. \((AI)\) for inequalities in the correct direction or brackets with values in the correct order or a clear word statement of the range. Follow through from their part (a).

[3 marks]

The graph of the quadratic function \( f(x) = 3 + 4x - x^2 \) intersects the \( y \)-axis at point A and has its vertex at point B.

8a. Find the coordinates of B.

**Markscheme**

\( x = -\frac{4}{-2} \) \( (M1) \)

\( x = 2 \) \( (AI) \)

OR

\( \frac{dy}{dx} = 4 - 2x \) \( (M1) \)

\( x = 2 \) \( (AI) \)

\( (2, 7) \) or \( x = 2, y = 7 \) \( (AI) \) \( (C3) \)

**Notes:** Award \((M1)(AI)(A0)\) for 2, 7 without parentheses.

[3 marks]

8b. Another point, \( C \), which lies on the graph of \( y = f(x) \) has the same \( y \)-coordinate as A.

(i) Plot and label \( C \) on the graph above.

(ii) Find the \( x \)-coordinate of \( C \).
9a. Write down the value of $c$.  

[1 mark]
9b. Find the value of \( b \).

**Markscheme**

\[
\frac{-b}{2(-1)} = 2 \quad (MI)
\]

Note: Award \((MI)\) for correct substitution in axis of symmetry formula.

OR

\[
y = 5 + bx - x^2
\]

\[
9 = 5 + b(2) - (2)^2 \quad (MI)
\]

Note: Award \((MI)\) for correct substitution of 9 and 2 into their quadratic equation.

\[
(b =)4 \quad (A1)(ft) \quad (C2)
\]

Note: Follow through from part (a).

9c. Find the \( x \)-intercepts of the graph of \( f \).

**Markscheme**

5, \(-1\) \((A1)(ft)(A1)(ft)\) \((C2)\)

Notes: Follow through from parts (a) and (b), irrespective of working shown.

9d. Write down \( f(x) \) in the form \( f(x) = -(x - p)(x + q) \).

**Markscheme**

\[
f(x) = -(x - 5)(x + 1) \quad (A1)(ft) \quad (CI)
\]

Notes: Follow through from part (c).
The front view of the edge of a water tank is drawn on a set of axes shown below. The edge is modelled by \( y = ax^2 + c \).

Point P has coordinates \((-3, 1.8)\), point O has coordinates \((0, 0)\) and point Q has coordinates \((3, 1.8)\).

10a. Write down the value of \(c\). \([1\ \text{mark}]\)

**Markscheme**

\[ 0 \quad (AI)(GI) \]

\([1\ \text{mark}]\)

10b. Find the value of \(a\). \([2\ \text{marks}]\)

**Markscheme**

\[ 1.8 = a(3)^2 + 0 \quad (MI) \]

OR

\[ 1.8 = a(-3)^2 + 0 \quad (MI) \]

**Note:** Award \((MI)\) for substitution of \(y = 1.8\) or \(x = 3\) and their value of \(c\) into equation. 0 may be implied.

\[ a = 0.2 \quad \left( \frac{1}{5} \right) \quad (AI)(ft)(GI) \]

**Note:** Follow through from their answer to part (a).

Award \((GI)\) for a correct answer only.

\([2\ \text{marks}]\)

10c. Hence write down the equation of the quadratic function which models the edge of the water tank. \([1\ \text{mark}]\)

**Markscheme**

\[ y = 0.2x^2 \quad (AI)(ft) \]

**Note:** Follow through from their answers to parts (a) and (b).

Answer must be an equation.

\([1\ \text{mark}]\)
10d. The water tank is shown below. It is partially filled with water.

Calculate the value of \( y \) when \( x = 2.4 \) m.

**Markscheme**

\[ 0.2 \times (2.4)^2 \quad (M1) \]

\[ = 1.15 \text{ (m)} \quad (A1)(G1) \]

**Notes:** Award (M1) for correctly substituted formula, (A1) for correct answer. Follow through from their answer to part (c).

Award (G1) for a correct answer only.

10e. The water tank is shown below. It is partially filled with water.

State what the value of \( x \) and the value of \( y \) represent for this water tank.

**Markscheme**

\( y \) is the height \((A1)\)

positive value of \( x \) is half the width \((or\ equivalent)\) \((A1)\)

[2 marks]
10f. The water tank is shown below. It is partially filled with water.

When the water tank is filled to a height of $0.9\, \text{m}$, the front cross-sectional area of the water is $2.55\, \text{m}^2$.

(i) Calculate the volume of water in the tank.

The total volume of the tank is $36\, \text{m}^3$.

(ii) Calculate the percentage of water in the tank.
The graph of the quadratic function \( f(x) = ax^2 + bx + c \) intersects the y-axis at point A \((0, 5)\) and has its vertex at point B \((4, 13)\).

11a. Write down the value of \( c \).

**Markscheme**

\[ 5 \text{ (AI) (C1)} \]  
[1 mark]

11b. By using the coordinates of the vertex, B, or otherwise, write down two equations in \( a \) and \( b \).

**Markscheme**

[3 marks]
Markscheme

at least one of the following equations required

\[ a(4)^2 + 4b + 5 = 13 \]
\[ 4 = \frac{b}{2a} \]
\[ a(8)^2 + 8b + 5 = 5 \]  \( \text{(A2)(AI)} \)  \( \text{(C3)} \)

**Note:** Award \( \text{(A2)(A0)} \) for one correct equation, or its equivalent, and \( \text{(C3)} \) for any two correct equations.

Follow through from part (a).

The equation \( a(0)^2 + b(0) = 5 \) earns no marks.

[3 marks]

11c. Find the value of \( a \) and of \( b \).

Markscheme

\[ a = -\frac{1}{2}, \quad b = 4 \]  \( \text{(A1)(ft)(AI)(ft)} \)  \( \text{(C2)} \)

**Note:** Follow through from their equations in part (b), but only if their equations lead to unique solutions for \( a \) and \( b \).

[2 marks]

The diagram below shows the graph of a quadratic function. The graph passes through the points \((6, 0)\) and \((p, 0)\). The maximum point has coordinates \((0.5, 30.25)\).

![Graph of a quadratic function](image)

12a. Calculate the value of \( p \).

Markscheme

\[ \frac{p+6}{2} = 0.5 \]  \( \text{(MI)} \)
\[ p = -5 \]  \( \text{(AI)} \)  \( \text{(C2)} \)

[2 marks]

12b. Given that the quadratic function has an equation \( y = -x^2 + bx + c \) where \( b, c \in \mathbb{Z} \), find \( b \) and \( c \).

[4 marks]
Markscheme

\[
\frac{-b}{2a} = 0.5 \quad (M1)
\]

\[b = 1 \quad (A1)\]

\[-0.5^2 + 0.5 + c = 30.25 \quad (M1)\]

\[c = 30 \quad (A1)(ft)\]

Note: Follow through from their value of \(b\).

OR

\[y = (6 - x)(5 + x) \quad (M1)\]

\[= 30 + x - x^2 \quad (A1)\]

\[b = 1, c = 30 \quad (A1)(A1)(ft) \quad (C4)\]

Note: Follow through from their value of \(p\) in part (a).

[4 marks]

The graph of a quadratic function \(y = f(x)\) is given below.

Write down the equation of the axis of symmetry.

[2 marks]

Markscheme

\[x = 3 \quad (A1)(A1) \quad (C2)\]

Notes: Award (A1) for \(x = \) (A1) for 3.

The mark for \(x = \) is not awarded unless a constant is seen on the other side of the equation.

[2 marks]
Markscheme

(3, −14)  (Accept \( x = 3, \ y = -14 \))  \((AI)(ft)(AI)\)  \((C2)\)

Note: Award \((AI)(A0)\) for missing coordinate brackets.

[2 marks]

13c. Write down the range of \( f(x) \).  \([2\text{ marks}]\)

Markscheme

\( y \geq -14 \)  \((AI)(AI)(ft)\)  \((C2)\)

Notes: Award \((AI)\) for \( y \geq \), \((AI)(ft)\) for \(-14\).
Accept alternative notation for intervals.

[2 marks]
The following curves are sketches of the graphs of the functions given below, but in a different order. Using your graphic display calculator, match the equations to the curves, writing your answers in the table below.

(the diagrams are not to scale)

<table>
<thead>
<tr>
<th></th>
<th>Function</th>
<th>Graph label</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>$y = x^3 + 1$</td>
<td>A</td>
</tr>
<tr>
<td>(ii)</td>
<td>$y = x^2 + 3$</td>
<td>B</td>
</tr>
<tr>
<td>(iii)</td>
<td>$y = 4 - x^2$</td>
<td>C</td>
</tr>
<tr>
<td>(iv)</td>
<td>$y = 2^x + 1$</td>
<td>D</td>
</tr>
<tr>
<td>(v)</td>
<td>$y = 3^{-x} + 1$</td>
<td>E</td>
</tr>
<tr>
<td>(vi)</td>
<td>$y = 8x - 2x^2 - x^3$</td>
<td>F</td>
</tr>
</tbody>
</table>
A quadratic curve with equation \( y = ax(x - b) \) is shown in the following diagram.

The \( x \)-intercepts are at (0, 0) and (6, 0), and the vertex \( V \) is at \((h, 8)\).

15a. Find the value of \( h \).  

**Markscheme**

\[
\frac{a + b}{2} = 3 \quad h = 3 \quad (MI)(AI) \quad (C2)
\]

**Note:** Award (MI) for any correct method.

**[2 marks]**

15b. Find the equation of the curve.

**[4 marks]**
16a. Factorise the expression \( x^2 - kx \).

\[ x(x - k) \quad (A1) \quad (C1) \]

[1 mark]

16b. Hence solve the equation \( x^2 - kx = 0 \).

\[ x = 0 \text{ or } x = k \quad (A1) \quad (C1) \]

Note: Both correct answers only.

[1 mark]
The diagram below shows the graph of the function \( f(x) = x^2 - kx \) for a particular value of \( k \).

16c. Write down the value of \( k \) for this function.

\[ f(x) = x^2 - kx \]

**Markscheme**

\[ k = 3 \quad (A1) \quad (C1) \]

[1 mark]

16d. The diagram below shows the graph of the function \( f(x) = x^2 - kx \) for a particular value of \( k \).

Find the minimum value of the function \( y = f(x) \).
Markscheme

Vertex at \( x = \frac{-(3)}{2(1)} \)  \( (MI) \)

Note: \( (MI) \) for correct substitution in formula.

\( x = 1.5 \)  \( (A1)(R) \)
\( y = -2.25 \)  \( (A1)(R) \)

OR

\( f'(x) = 2x - 3 \)  \( (MI) \)

Note: \( (MI) \) for correct differentiation.

\( x = 1.5 \)  \( (A1)(R) \)
\( y = -2.25 \)  \( (A1)(R) \)

OR

for finding the midpoint of their 0 and 3  \( (MI) \)
\( x = 1.5 \)  \( (A1)(R) \)
\( y = -2.25 \)  \( (A1)(R) \)

Note: If final answer is given as \((1.5, -2.25)\) award a maximum of \((MI)(A1)(A0)\)

[3 marks]